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Comment on "Direct Solutions for Strum-Liouville Systems with Discontinuous Coefficients"

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HODGES[1,2] has presented a powerful technique for solving this class of problems based upon using separate modal series for each interval within which the otherwise discontinuous coefficients are smoothly varying functions. The method is basically a Ritz approach using simple polynomials in the modal series and (geometric) constraint conditions to enforce the proper physical conditions between intervals. These constraint conditions are used to eliminate certain of the modal coefficients in the series. One thus obtains a reduced problem of the usual modal type whose order is that of the total number of terms in all the modal series minus the number of constraints.

The purpose here is to note that problems of this type have been treated inter alia in the context of component mode analyses and to discuss some of the options the analyst has in considering such problems. The reader's attention is drawn to Refs. 3 and 4, the former being most directly relevant to the present discussion and the latter containing an extensive references list of broadly related literature.

In the approach of Klein,[3] modal expansions are again made in each interval. However free-free normal modes rather than polynomials are used. These normal modes (which are known analytically for uniform beams) are convenient because of their orthogonality properties, but they do lack the basic simplicity of polynomials. For structures where the free-free normal modes are not readily known (e.g., plates rather than beams), products of beam free-free modes have been proven effective in similar analyses.[5] Of course, one can use polynomials or other primitive modes to compute the free-free normal modes for a given interval.

Another difference between Refs. 1, 2, 3, and 4 is in the use of the constraint conditions. In Refs. 1 and 2 these are used to eliminate a number of modal coefficients equal to the number of constraints. By contrast, in Ref. 3 the constraints are taken into account by using Lagrange multipliers, thereby retaining all modal coefficients. The latter procedure increases (at least temporarily, but see remarks below) the total number of unknowns to the number of all original modal coefficients plus a number of Lagrange multipliers equal to the number of constraint conditions. It has the advantage of treating all modal coefficients on an equal footing and also avoids any theoretical possibility of forming an ill-conditioned matrix by an inadvertent choice of modal coefficients for elimination by the constraint conditions. The Lagrange multipliers also supply physical information, i.e., the forces of constraint. The eigenvalue problem resulting from the use of Lagrange multipliers as in Ref. 3 has been solved successfully directly.[6] However, as Klein[3] and Dowell[4] have emphasized, because of the orthogonality (in each interval) of the assumed free-free normal modes, one may eliminate analytically all of the modal coefficients by solving for them in terms of the Lagrange multipliers. This reduces the order of the eigenvalue problem to one equal to the number of constraints. The reduced eigenvalue matrix is not of the usual form (having poles at each natural frequency of each free-free component), but it is readily solved numerically. It also provides a means of obtaining bounds on the eigenvalues of the total system.[4]

Whether the analyst will find the method of Refs. 1 and/or 3 and 4 more effective for a given application is a highly subjective decision. Perhaps the point which should be emphasized (as Hodges[1,2] and Klein[3] have noted) is that Refs. 1, 2, and 3 provide a method of generating finite elements of variable order by selecting a variable number of modes in each interval. They thereby produce a highly flexible and versatile extension to the finite element method. In particular, this method generates in a very efficient manner higher order finite elements, which allow the use of a small number of elements (components) in regions with smoothly varying system properties. Such elements provide more accurate solutions than a larger number of low order elements. Indeed, for the former convergence is always assured as the order of the element increases, while for the latter convergence is often much slower and sometimes nonexistent. In general, one wishes to use the smallest number of elements possible which accurately represents the variation of system properties such as mass and stiffness. The approach of Ref. 3 is advantageous in this respect in that the cost of computation is determined primarily by the number of elements, and is virtually independent of the number of degrees of freedom per element.

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Reply by Author to Earl H. Dowell

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I APPRECIATE Prof. Dowell's interest and comments on Refs. 1 and 2 and have only three points to add. First, there is additional complexity in the analysis of general structures by element (i.e., component) normal modes with Lagrange multipliers.[3] Unfortunately, this increase in complexity does

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